Modeling with Quadratic Functions Video Lecture

Section 3.4

Course Learning Objectives:

1) Graph polynomial functions and use such graphs to solve applied problems and to understand the significance of attributes of the graph to such applied problems.
2) Solve appropriate applications of determining the maximum or minimum of a quadratic function.
3) Identify and articulate the significance of graphical components in a mathematical model/application.

Weekly Learning Objectives:

1) Build function models from verbal descriptions.
2) Determine exact values of maximums or minimums in quadratic application problems.
3) Use a graphing utility to approximate maximum or minimum values in application problems.

Modeling with Quadratic Functions

The height of a cone is twice as long as its radius. Find a function that models the cone's volume V in terms of its height h.

Express the area of a circle as a function of its circumference.

Guidelines for Modeling with Functions:

1. Express the Model in Words. Identify the quantity you want to model and express it, in words, as a function of the other quantities in the problem.
2. Choose the Variable. Identify all the variables used to express the function in step 1. Assign a symbol to one variable and express the other variables in terms of this symbol.
3. Create a Function. Express the function using only the one variable chosen.
4. Solve. Use the function to answer the questions posed in the problem. (To find a maximum or minimum of a quadratic function, use the algebraic vertex formula or graphical methods.)

A rancher with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. Find the largest possible total area of the four pens.

Find the dimensions of the rectangle with the largest area having one side on the positive x-axis, one side on the positive y-axis and its upper right corner on the line $3x + 6y = 6$.

A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.

A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides.

a) Find the largest volume that such a box can have.
b) Find the values of x for which the volume is greater than 200 in^3.

A wire 10 meters long is to be cut into two pieces. One piece will be shaped as a square, and the other piece will be shaped as a circle.

a) Find a function that models the total area A in terms of the length, x, of the piece of wire bent into a circle.
b) Find the value of x that would minimize the total area.

An island is 2 miles from the nearest point P on a straight shoreline. A town is 12 miles down the shore from point P. Assume a person can row a boat an average speed of 3 miles per hour and the same person can walk 5 miles per hour.

a) Find a function that models the time T needed for the trip to get from the island to town.
b) Where should the person land so that he reaches the town as soon as possible?

A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 200,000 square meters in order to provide enough grass for the herd.

a) What dimensions would require the least amount of fencing if no fencing is needed along the river?

b) Now assume that the farmer has decided to put a special wire fence along the river side and less expensive fencing along the other three sides. The cheaper fencing costs $2 per meter and the more expensive fencing along the river side costs $5 per meter. Determine a cost function and find the dimensions that will minimize the cost of the fencing.

c) If the farmer has at most $6000 to spend on fencing, find the range of lengths he can fence along the river to stay within budget.