

Addition and Subtraction Formulas

$$\begin{aligned}\sin(x + y) &= \sin x \cos y + \cos x \sin y \\ \sin(x - y) &= \sin x \cos y - \cos x \sin y\end{aligned}$$

$$\begin{aligned}\cos(x + y) &= \cos x \cos y - \sin x \sin y \\ \cos(x - y) &= \cos x \cos y + \sin x \sin y\end{aligned}$$

$$\begin{aligned}\tan(x + y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y} \\ \tan(x - y) &= \frac{\tan x - \tan y}{1 + \tan x \tan y}\end{aligned}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x\end{aligned}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Formulas for Lowering Powers

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Half-Angle Formulas

$$\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Product - to - Sum Formulas

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\cos x \sin y = \frac{1}{2}[\sin(x + y) - \sin(x - y)]$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$