

The Zeros of a Polynomial Video Lecture

Sections 4.2 and 4.3

Course Learning Objectives:

- 1) Graph polynomial functions and use such graphs to solve applied problems and to understand the significance of attributes of the graph to such applied problems.**
- 2) Solve appropriate applications of determining zeros of polynomials.**

Weekly Learning Objectives:

- 1) Use the rational zeros theorem to list the potential real zeros of a polynomial function.**
- 2) Find the exact value of all zeros of a polynomial function.**
- 3) Factor polynomials completely.**
- 4) Use the Boundedness Theorem.**

The Zeros of a Polynomial

Theorem: A polynomial function of degree n ($n \geq 1$), has at most n real zeros.
(We count each zero as many times as its multiplicity.)

$$P(x) = x^3 - x^2 - 14x + 24 = (x-3)(x-2)(x+4)$$

The following theorem gives us a list of possible rational zeros for a polynomial function if the coefficients are integers.

Rational Zeros Theorem: If the polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

has integer coefficients, then every rational zero of $P(x)$ is of the form $\frac{p}{q}$ where p is a factor of a_0 and q is a factor of a_n .

List the possible rational zeros for $f(x) = 2x^3 + x^2 - 13x + 6$
Find all the real zeros.

List the possible rational zeros for $f(x) = x^4 - 5x^3 - 5x^2 + 23x + 10$
Find the real zeros.

Boundedness Theorem: Let $P(x)$ be a polynomial with real coefficients.

1. If we divide $P(x)$ by $x - b$ using synthetic division, and if the row that contains the quotients and the remainder has no negative entry, then b is an upper bound for the real roots of $P(x) = 0$.
2. If we divide $P(x)$ by $x - b$ using synthetic division, and if the row that contains the quotients and the remainder has entries that are alternately nonpositive and nonnegative then b is a lower bound for the real roots of $P(x) = 0$.

Show that -3 and 5 are the lower and upper bounds for the real roots of

$$x^4 - 2x^3 - 9x^2 + 2x + 8 = 0$$

Find the exact value of all zeros of $g(x) = 7x^3 - 32x^2 + 44x - 16$

Find the exact value of all zeros of $f(x) = x^4 - 6x^3 + 4x^2 + 15x + 4$

Find the exact value of all zeros of $g(x) = 2x^6 - 3x^5 - 13x^4 + 29x^3 - 27x^2 + 32x - 12$

We know that n th degree polynomial can have at most n real zeros. In the complex number system an n th degree polynomial has exactly n zeros (counting multiplicity). This is called the **Fundamental Theorem of Algebra**.

Find all zeros of $P(x) = x^3 - 8$ and factor it completely.

Find all zeros of $P(x) = x^4 + 10x^2 + 25$ and factor it completely.

Find all zeros of $P(x) = x^3 - x^2 + x$ and factor it completely.

Find all zeros of $P(x) = 3x^5 + 24x^3 + 48x$ and factor it completely.

Find all zeros of $P(x) = x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48$ and factor it completely.

Find all zeros of $P(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$ and factor it completely.