Sum and Difference Formulas Video Lecture

Section 7.5

Course Learning Objectives:

1) Demonstrate an understanding of trigonometric functions and their applications.
2) Verify identities.

Weekly Learning Objectives:

1) Use sum and difference formulas to find exact values.
2) Use sum and difference formulas to establish identities.
3) Write expressions of the form Asinx + Bcosx as a single sine function.

Sum and Difference Formulas

We will derive identities involving the trigonometric functions of the sum and difference of numbers/angles.

Recall that $\cos(\alpha+\beta) \neq \cos\alpha + \cos\beta$. Convince yourself by choosing a value for α and a value for β that will show that $\cos(\alpha+\beta) \neq \cos\alpha + \cos\beta$.

In general it is true that $\textbf{trig}(a \pm b) \neq \textbf{trig}\,(a) \pm \textbf{trig}\,(b)$

Derivation of identity for $\cos(\alpha+\beta)$

$P_0\,(1,0)$ $\quad$ $Q_0\,(\cos(-\alpha), \sin(-\alpha))$

$P_1\,(\cos(\alpha+\beta), \sin(\alpha+\beta))$ $\quad$ $Q_1\,(\cos\beta, \sin\beta)$

$\cos(\alpha - \beta) =$

$\sin(\alpha + \beta)$

$\sin(\alpha - \beta)$

$\tan(\alpha + \beta)$

$\tan(\alpha - \beta)$

The following identities need to be memorized.

Sum & Difference Formulas

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Examples:

1. Find the exact value for $\sin\left(\frac{\pi}{12}\right)$.

2. Find the exact value for $\cos 105°$.

3. Find the exact value for $\sec\left(\frac{7\pi}{12}\right)$.

4. Find the exact value for $\cos 40° \cos 10° + \sin 40° \sin 10°$.

5. Prove the identities

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \text{and} \qquad \sin\left(\frac{\pi}{2} - u\right) = \cos u$$

6. Prove the identity $\cos(x + y) + \cos(x - y) = 2\cos x \cos y$.

7. Prove the identity $\cos\left(\frac{3\pi}{2} + \theta\right) = \sin\theta$

8. If $f(x) = \sin x$, show that

$$\frac{f(x+h) - f(x)}{h} = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)$$

Write an expression in terms of sine for $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$.

In general, if A and B are real numbers, then

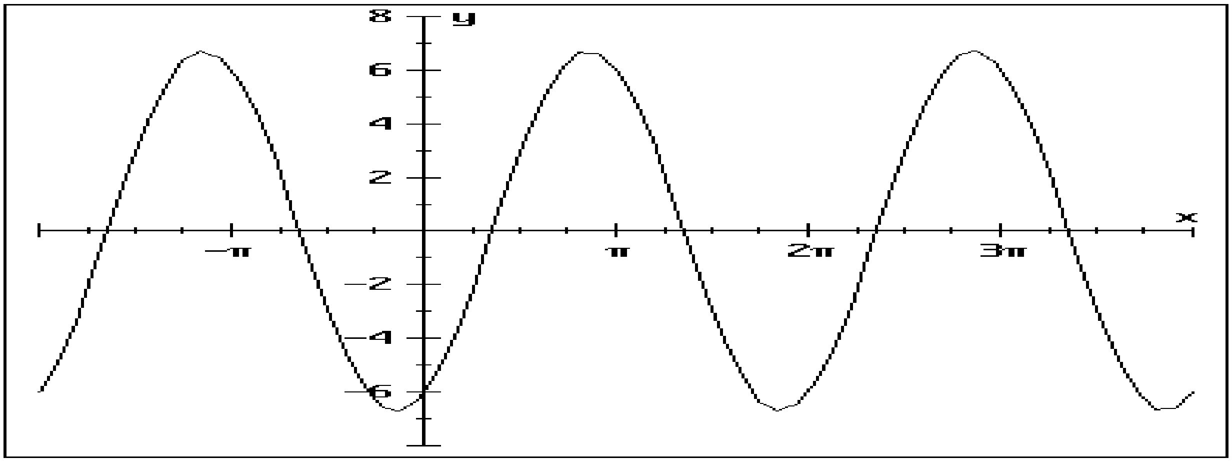
$$A\sin x + B\cos x = k\sin(x+\phi) \text{ where } k = \sqrt{A^2+B^2} \text{ and}$$

$$\phi \text{ satisfies } \cos\phi = \frac{A}{\sqrt{A^2+B^2}} \text{ and } \sin\phi = \frac{B}{\sqrt{A^2+B^2}}$$

Proof:

Examples:

9. The following is a graph $f(x) = 3\sin x - 6\cos x$ using technology.



Does it look like a variation of the sine function?

What is the period? Amplitude? Phase Shift?

10. Write the expression $3\sin x - 6\cos x$ in terms of the sine only

Regarding the function in example 9,

What is the period? Amplitude? Phase Shift?