

Related Rates

In many practical applications of Calculus, several related quantities vary together. Therefore, the rates at which they vary are also related to each other. The generic related rates problem asks for one rate of change, given information about one or more others.

The idea behind solving related rates problems is the same: start with an equation relating two or more quantities that vary with time. Differentiate both sides with respect to time using the Chain Rule. The resulting equation relates the rates at which the quantities vary. Use it to solve for the desired rate.

Taking derivatives with respect to t in Related Rates problems!

$$\frac{d}{dx}(x^2) = \qquad \frac{d}{dx}(y^2) =$$

$$\frac{d}{dt}(t^2) = \qquad \frac{d}{dt}(x^2) = \qquad \frac{d}{dt}(y^2) =$$

Procedure for Solving Related Rates Problems:

- 1) Assign variables to all given quantities and unknown quantities. Make a sketch and label.
- 2) Write an equation involving the variables whose rates of change are given or are to be determined.
- 3) Use the Chain Rule to differentiate implicitly on both sides with respect to time.
- 4) Substitute in all known values and solve for the missing amount.

3. A 10 foot ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?
4. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of $2 \text{ m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3 m deep.

5. Car A is going west at 50 mph and car B is going north at 60 mph. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 miles and car B is 0.4 miles from the intersection?
6. A person 6 ft tall is walking away from a streetlight 20 ft high at the rate of 7 ft/s. At what rate is the length of the person's shadow increasing?

7. A fishing line is reeled in at a rate of 1 foot per second from a bridge 15 feet above the water. At what rate is the angle between the line and the water changing when 25 feet of line is out?
8. A patrol car is parked 50 feet from a long warehouse. The revolving light on top of the car turns at a rate of 30 revolutions per minute. How fast is the light beam moving along the wall when the angle between the perpendicular line from the patrol light to wall and the line from the patrol light to where the beam meets the wall is 30° .