Parametric Equations Video Lecture

Section 10.7

Course Learning Objectives:
Graph parametric equations with and without technology.

Weekly Learning Objectives:

1) Graph parametric equations by hand.
2) Graph parametric equations using a graphing utility.
3) Find a rectangular equation for a curve defined parametrically.
4) Use time as a parameter in parametric equations.
5) Find parametric equations for curves defined by rectangular equations.
6) Find the orientation of a curve defined parametrically.
7) Solve applications involving parametric equations.

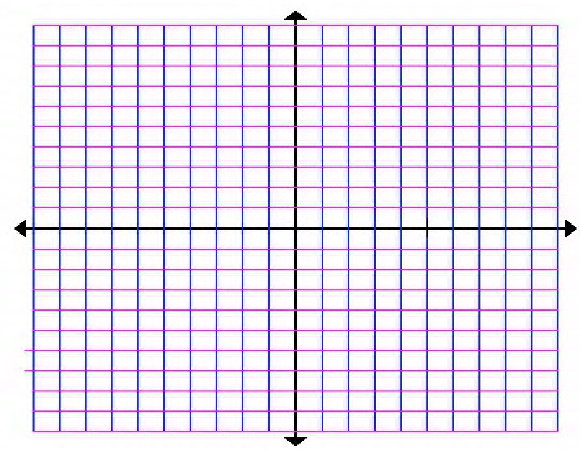
Plane curves and Parametric Equations

In this lecture we will discuss graphs as a collection of ordered pairs (x, y), which may or may not be a function, and which are defined by what we call parametric equations. Parametric equations are often used to describe movement along a curve with respect to a variable t, where t may represent time.

Let $x = f(t)$ and $y = g(t)$ where f and g are two functions whose common domain is some interval I. The collection of points defined by $(x, y) = (f(t), g(t))$ is called a plane curve and the equations $x = f(t)$ and $y = g(t)$, where t is in I, are called parametric equations of the curve. The variable t is called a parameter. When plotting the points for increasing values of t a direction for the curve is established. This is called the orientation of the graph.

1. Consider the parametric equations $x = 1 - 2t$ and $y = 1 + t$, $-1 \leq t \leq 4$. Complete the table and sketch the graph indicating the orientation.

t	-1	0	1	2	3	4
x						
y						



Note: As t increases, x and y

Sketch the graph on the calculator:

Eliminate the parameter:

2. Sketch the graph of the parametric equations $x = t^3$ and $y = t^2$, t ϵ R noting the orientation.

3. Sketch the graph of the parametric equations $x = \cos t - 2$ and $y = \sin t + 3$ noting the orientation of the graph.

Eliminate the parameter

4. Each of the following sets of parametric equations have the same rectangular form but different graphs. Sketch the graph of each and note the restrictions and the orientation.

a) $x = t$ and $y = 1 - t$

Restrictions:

Orientation:

Eliminate the parameter:

b) $x = 1 - t^2$ and $y = t^2$

Restrictions:

Orientation:

Eliminate the parameter:

c) $x = \cos^2 t$ and $y = \sin^2 t$

Restrictions:

Orientation:

Eliminate the parameter:

5. Write two different parametric equations for each rectangular equation.

$y = x^2 - 1$ $\qquad\qquad$ $y = \sqrt{x}$

6. Find parametric equations for an object that moves along the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ with the motion described:

a. The motion begins at $(2, 0)$, is counterclockwise, and requires 2π seconds for a complete revolution.

b. The motion begins at $(2, 0)$, is counterclockwise, and requires 1 second for a complete revolution.

c. The motion begins at $(0, 3)$, is clockwise, and requires 1 second for a complete revolution.

7. Write a rectangular equation for the following parametric equation:
$x = h + a \sec\theta,\ y = k + b\tan\theta, \theta$ is the parameter.

Projectile motion and simulated motion using time as a parameter:

Consider a projectile launched at a height h feet above the ground and at an angle θ to the horizontal. If the initial speed is v_0 feet per second, the path of the projectile is modeled by the parametric equations:

$$x = (v_0 \cos\theta)t \qquad y = -16t^2 + (v_0 \sin\theta)t + h.$$

8. Suppose that Jim hit a golf ball with an initial velocity of 150 feet per second at an angle of $30°$ to the horizontal.

a. Find parametric equations that describe the position of the ball as a function of time.

b. How long is the ball in the air?

c. When is the ball at its maximum height? Determine the maximum height of the ball.

d. Determine the distance the ball traveled.

e. Graph to view the motion.