

Law of Sines Video Lecture

Section 8.2

Course Learning Objectives:

Demonstrate an understanding of trigonometric functions and their applications.

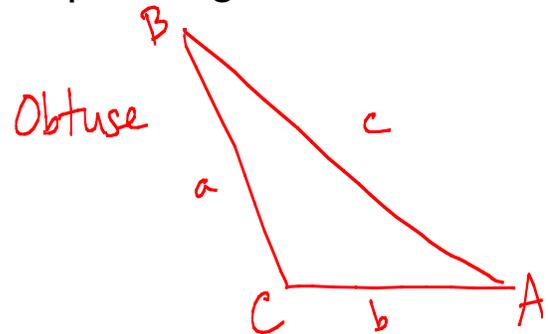
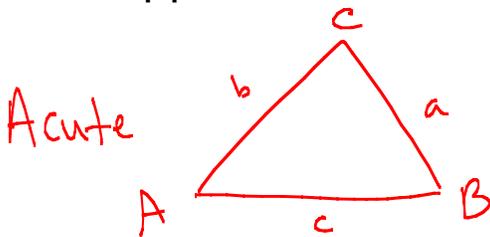
Weekly Learning Objectives:

- 1) Solve SAA or ASA triangles.**
- 2) Solve SSA triangles.**
- 3) Solve applied problems.**

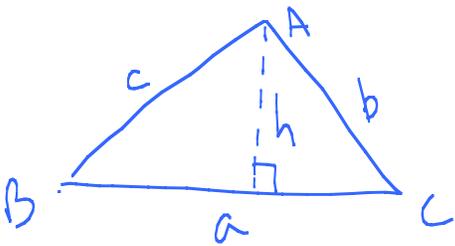
The Law of Sines

We have used trigonometric functions to "solve" right triangles. We will now learn how the trigonometric functions can be used to solve oblique (no right angles) triangles. There are two relationships that can be used depending upon the information that is known about the triangle. In this section we discuss the Law of Sines which says that the lengths of sides in a triangle are proportional to the sines of the angles opposite these sides.

An oblique triangle may be acute or obtuse as shown in the diagrams below. We will continue to use the standard labeling of naming the sides as lower case letters and the vertices/angles as the upper case letter opposite the corresponding side.



Proof of the Law of Sines:

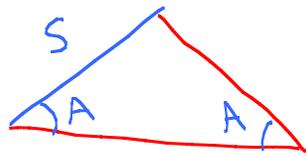


In general a triangle can be "solved" if we know three of the six parts (except for knowing the three angles because this does not determine a unique triangle). The various possibilities are

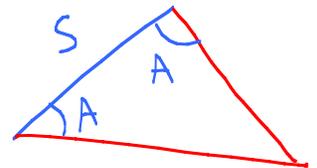
- 1) SAA or ASA One side and two angles
- 2) SSA Two sides and the angle opposite one of those sides
- 3) SAS Two sides and the included angle
- 4) SSS Three sides

Diagrams for the four cases:

1) SAA



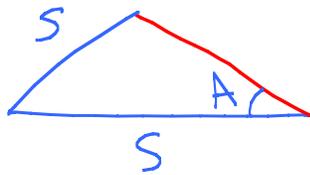
or 1) ASA



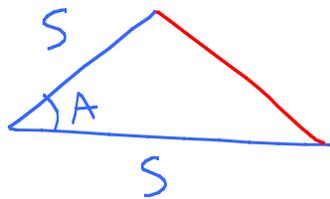
Use Law of Sines

2) SSA

ambiguous case

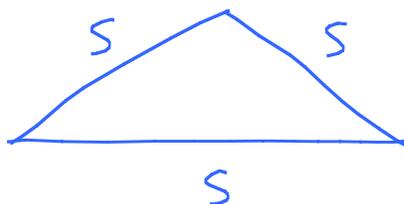


3) SAS



Use Law of Cosines

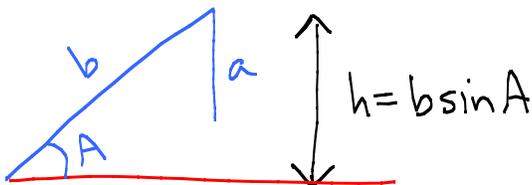
4) SSS



You can see that a unique triangle is formed in these situations. However, in the second case there may be 2 triangles with the given properties, 1 triangle, or no triangles. For this reason, this situation is referred to as the ambiguous case. Sketch various possibilities showing how you can get 0, 1, or 2 triangles in case 2.

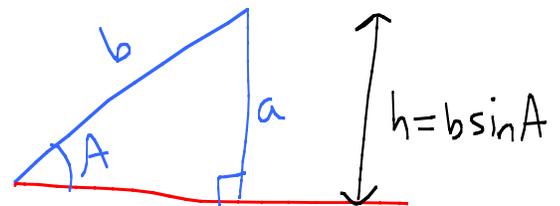
NO TRIANGLE

if $a < h = b \sin A$, then a is not long enough to form a triangle



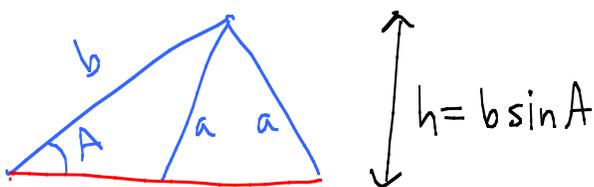
ONE RIGHT TRIANGLE

if $a = h = b \sin A$, then it forms a right triangle



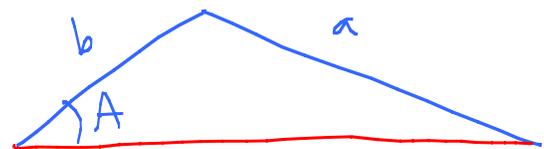
TWO TRIANGLES

if $h = b \sin A < a < b$, then 2 distinct triangles can be formed



ONE TRIANGLE

if $a \geq b$, then only one triangle can be formed



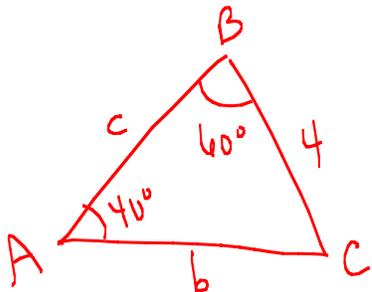
We do not need to rely on the sketch to determine which situation we are in. **In general, whenever you are finding the second angle, if the $\sin A < 1$, we must check the angle and its supplement as possibilities, because any angle smaller than 180° can be in the triangle.**

Remember all 3 angles must add 180° .

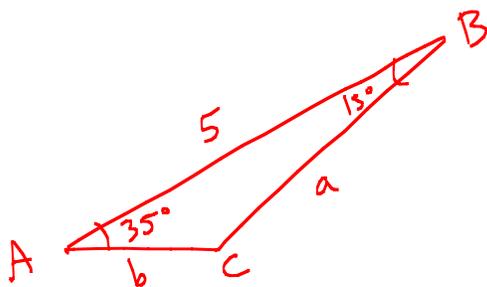
Examples: Solve each triangle.

SAA or ASA case:

1) $A = 40^\circ$, $B = 60^\circ$, $a = 4$

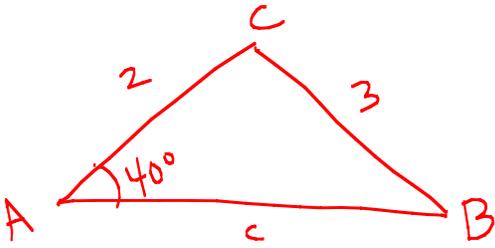


2) $A = 35^\circ$, $B = 15^\circ$, $c = 5$

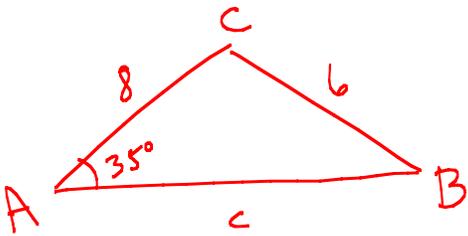


SSA case (ambiguous case):

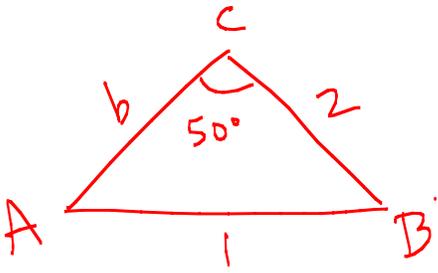
3) $a = 3, b = 2, A = 40^\circ$



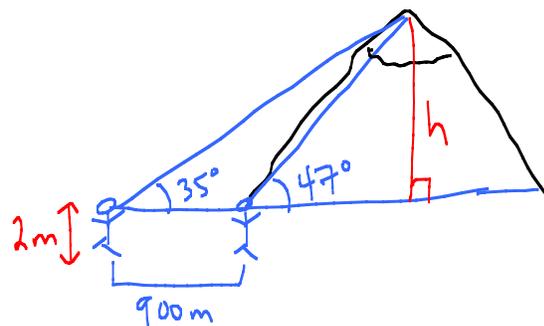
4) $a = 6, b = 8, A = 35^\circ$



5) $a = 2, c = 1, C = 50^\circ$



- 6) To measure the height of a mountain, a surveyor takes two sightings of the peak at a distance of 900 meters apart on a direct line to the mountain. The first observation results in an angle of elevation of 47° , and the second results in an angle of elevation of 35° . If the transit is 2 meters high, what is the height h of the mountain?



7) The navigator of a ship at sea spots two lighthouses that she knows are to be 3 miles apart along a straight seashore. She determines that the angles formed between two line-of-sight observations of the lighthouses and the line from the ship directly to shore are 15° and 35° respectively.

a) How far is the ship from lighthouse A?

b) How far is the ship from lighthouse B?

c) How far is the ship from shore?