Integration of Inverse Trigonometric Functions

Integrals Involving Inverse Trigonometric Functions:

Let u be a differentiable function of x, and let $a > 0$.

1. $\int \frac{du}{\sqrt{a^2-u^2}} = arcsin\frac{u}{a} + C$

2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}arctan\frac{u}{a} + C$

3. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}arcsec\frac{|u|}{a} + C$

These can be proven by differentiation.
It is only necessary to remember these three formulas, since the alternative would just use negatives. For example,

$$\int \frac{du}{\sqrt{a^2-u^2}} = arcsin\frac{u}{a} + C \quad \text{or} \quad -arccos\frac{u}{a} + C$$

1. $\int \frac{3}{\sqrt{1-4x^2}}dx$

2. $\int \frac{t}{t^4+16}dt$

3. $\int \frac{dx}{\sqrt{e^{2x}-1}}$

4. $\int \frac{4x+3}{\sqrt{1-x^2}} dx$

5. $\int \frac{3}{2\sqrt{x}(1+x)} dx$

6. $\int_0^{\frac{\pi}{2}} \frac{cosx}{1+sin^2x} dx$

7. $\int \frac{2x-5}{x^2+2x+2} dx$

8. $\int \frac{x}{\sqrt{9+8x^2-x^4}}dx$

9. $\int \frac{\sqrt{x-2}}{x+1} dx$

10. Find the area under $y = \frac{e^x}{1+e^{2x}}$ for x in $[0, ln\sqrt{3}]$

11. Determine which of the following integrals can be found using the basic integration formulas that we have learned so far:

a. $\int \frac{dx}{x\sqrt{x^2-1}}$

b. $\int \frac{xdx}{\sqrt{x^2-1}}$

c. $\int \frac{dx}{\sqrt{x^2-1}}$

d. $\int \frac{dx}{xlnx}$

e. $\int \frac{lnxdx}{x}$

f. $\int lnxdx$