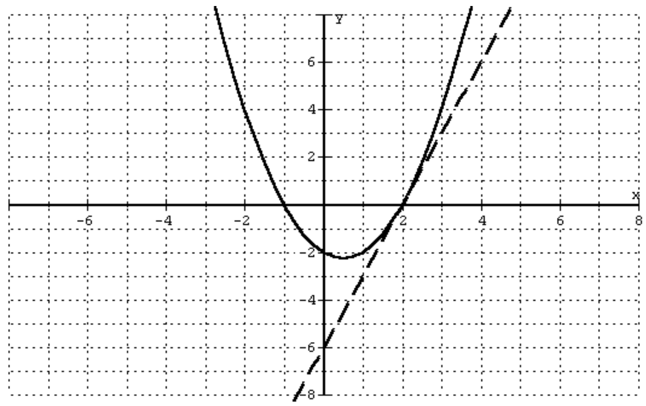
# Differentials

We want to find values of a function for arguments close to $c$.

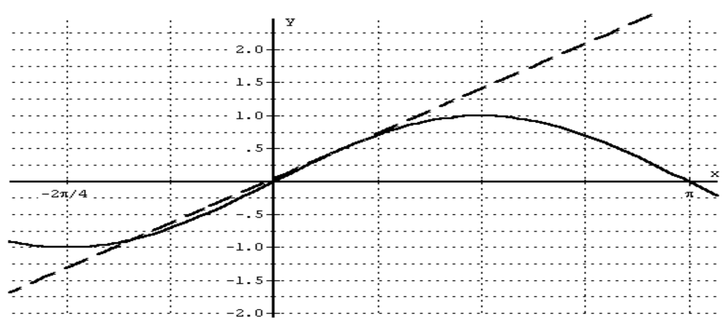


Use the $y$-coordinate on the tangent line as an approximation.

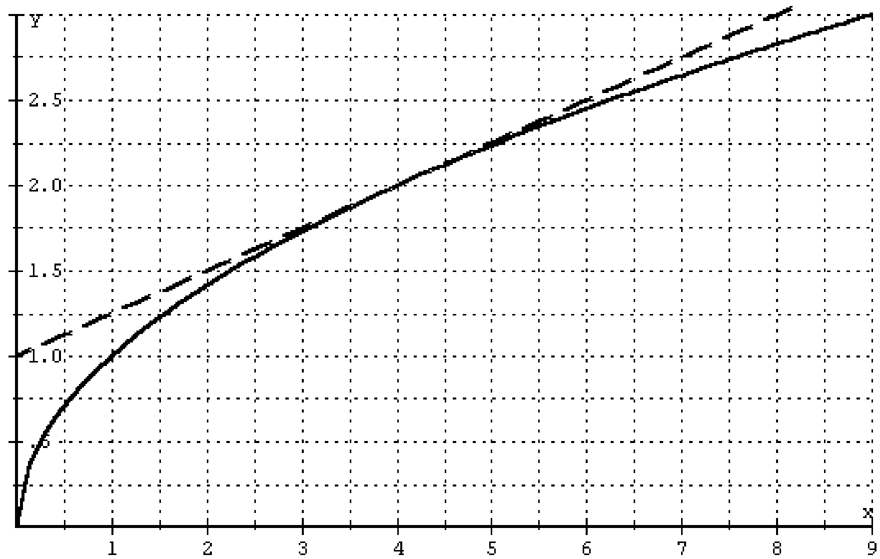
Equation of tangent line: $y - f(c) = f'(c)(x - c)$

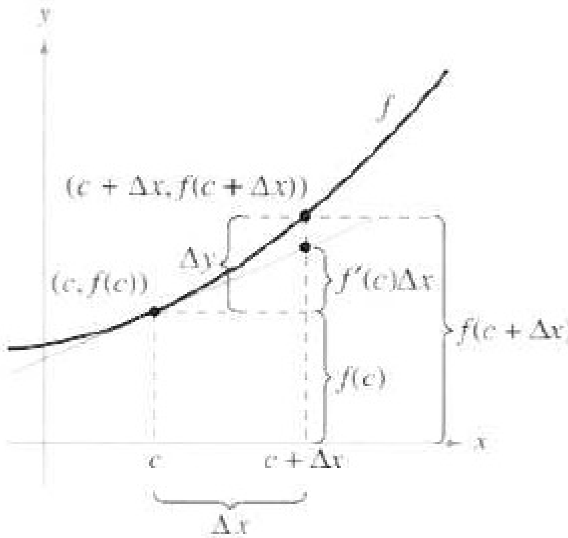
**Tangent Line Approximation to $f(x)$ : $y = f'(c)(x - c) + f(c)$**
**(Picking $x$'s sufficiently close to $c$ should give $y-$ values that can be used as approximations to the true value of $f(x)$).**

**1.** Approximate $\sin 31°$ using a tangent line.



**2.** Approximate $\sqrt{4.2}$ using a tangent line.






Definitions:

When the tangent line to the graph of $f$ at the point $(c, f(c))$

$$y = f(c) + f'(c)(x - c)$$

is used as an approximation of the graph of $f$, the quantity $x - c$ is called the change in $x$, and is denoted by $\Delta x$. When $\Delta x$ is small, the change in $y$ (called $\Delta y$) can be approximated by:

$$\begin{aligned}\Delta y &= f(c + \Delta x) - f(c) \\ &\approx f'(c)\Delta x\end{aligned}$$

For such an approximation, the quantity $\Delta x$ is denoted by $dx$, and is called the **differential of $x$**. The expression $f'(x)dx$ is denoted by $dy$, and is called the **differential of $y$**.

**Differential of $x$ = $\Delta x$ or $dx$**

**Differential of $y$ = $dy$** $\qquad$ $\boldsymbol{dy = f'(x)dx}$

**In many applications, the differential of $y$ can be used as an approximation of the change in $y$:**

$\boldsymbol{\Delta y \approx dy}$ **for small $\Delta x$** $\qquad$ $\boldsymbol{\Delta y \approx f'(x)dx}$

**The actual change in $y$ would be:**

$$\boldsymbol{\Delta y = f(x + \Delta x) - f(x)}$$

$\Delta x$ or $dx$ is the horizontal change as you move from one point to another along a tangent line OR along the curve.

$\Delta y$ is the vertical change as you move from one point to another along the curve.

$dy$ is the vertical change as you move from one point to another along the tangent line.

**Formulas:**

**Differential of $y$:** $dy = f'(x)dx$

**Actual change in $y$:**
$\Delta y = f(x + \Delta x) - f(x)$

**Approximate change in $y$:**
$dy = f'(x)dx$

**Actual $y-$ value:**
$f(x + \Delta x)$

**Approximate $y-$ value:**
$f(x) + f'(x)dx$ **(using the tangent line)**

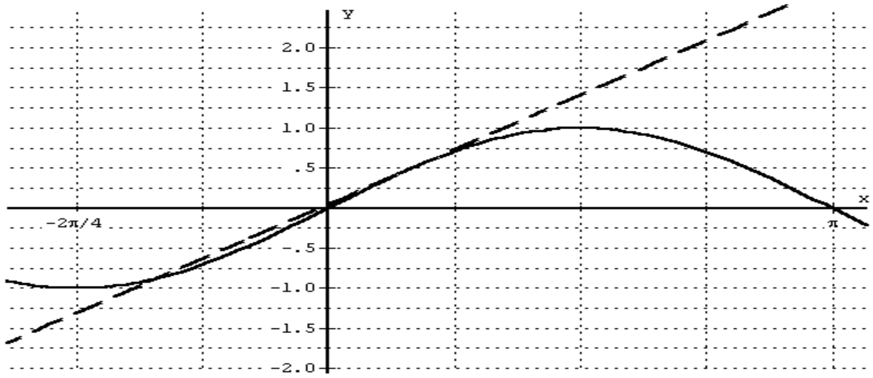
**Finding Differential Form:**

**3**. Find $dy$ if

a) $y = \sqrt{x^2 - 4}$

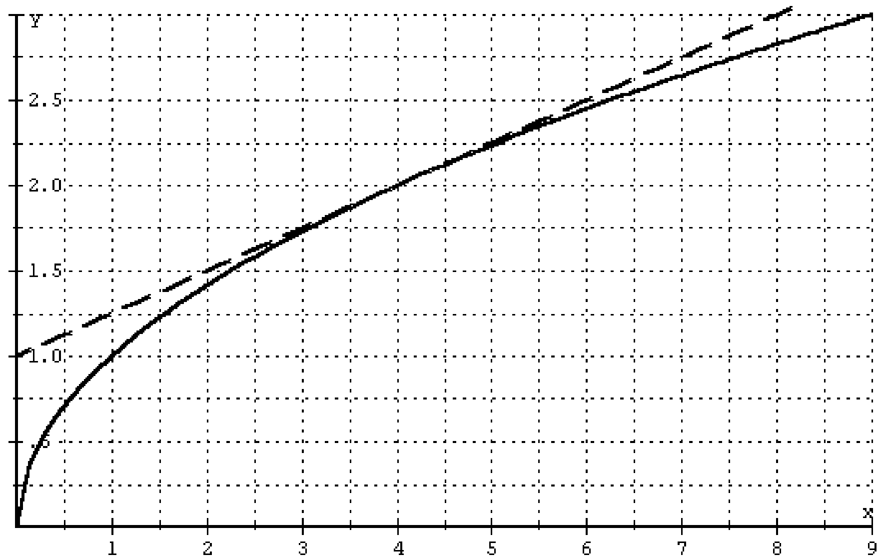
b) $y = x \sin x$

**4.** Approximate $\sin 31°$ using a differential approximation.




$$f(x + \Delta x) \approx f(x) + f'(x)dx$$

**5**. Approximate $\sqrt{4.2}$ using a differential approximation.




$$f(x + \Delta x) \approx f(x) + f'(x)dx$$

**6.** If $y = 1 - x^2$, find and compare $\Delta y$ with $dy$ when $x = 1$ and $\Delta x = dx = -.1$ Sketch a graph and anticipate the results first.

**7.** Approximate $\sqrt[3]{7.9}$ using:

a) Differentials

b) Tangent Line Approximation

| | | | | |
|---|---|---|---|---|
| $\Delta y$ | $=$ | $f(x + \Delta x)$ | $-$ | $f(x)$ |
| $Propagated\,Error$ | | $Exact\,Value$ | | $Measured\,Value$ |

**8.** The measurement of the edge of a cube is found to be $12$ inches, with a possible error of $.03$ inches.

a) Use differentials to approximate the maximum possible error in computing the volume of the cube.
b) Also find the actual propagated error and compare.
c) Is the propogated error large or small?
Find the relative error $\frac{\Delta V}{V}$.

**9**. Use differentials to approximate the exterior volume of a spherical shell given that the thickness of the shell is $.2$ cm and the inside radius of the sphere is $100$ cm. Also calculate the exact volume of the shell.

**10**. The measurement of the circumference of a circle is found to be $56$ centimeters. Approximate the percentage error in computing the area of the circle if the possible error in measuring the circumference is $1.2$ cm.

Note: Percentage (Relative) Error is $\frac{\Delta A}{A}$ which can be approximated by $\frac{dA}{A}$

**11.** Let $f(x) = \sqrt{x}$

a) Give a linear approximation to $f(x)$ at $x = 9$.

b) Use your approximation to approximate $f(8.9)$.

c) Find the differential $dy$ for $x = 9,\ \Delta x = -.1$

**12.** Let $f(2) = 6$ and $f'(2) = -1$.

a) Give a linear approximation to $f(x)$ at $x = 2$.

b) Use your approximation to approximate $f(2.1)$.

c) Find the differential $dy$ for $x = 2,\ \Delta x = .1$