Derivatives of Inverse Functions

Theorem: Let f be a function whose domain is an interval I. If f has an inverse function, then the following statements are true.

1. If f is continuous on its domain, then f^{-1} is continuous on its domain.

2. If f is differentiable at c and $f'(c) \neq 0$, then $f^{-1}$is differentiable at $f(c)$.

Theorem: Let f be a function that is differentiable on an interval I. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$. Moreover,

$$g'(x) = \frac{1}{f'(g(x))}, f'(g(x)) \neq 0$$

1. Let $f(x) = x^5 - x^3 + 2x$.
   Let $g(x)$ be the inverse function of $f(x)$.
   a. What is the value of $g(x)$ when $x = 2$?

   b. What is the value of $(g)'(x)$ when $x = 2$?

The reciprocal relationship shown in example 1 is sometimes written as: $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

2. Let $f(x) = tanx, \ -\frac{\pi}{2} < x < \frac{\pi}{2}$
   Find the derivative of the inverse tangent function.

Note: This shows that the derivative of an inverse trig function is an algebraic function. Just like the derivative of the transcendental function $f(x) = lnx$ is an algebraic function $f'(x) = \frac{1}{x}$.

Derivatives of Inverse Trigonometric Functions:
Let u be a differentiable function of x.

$\frac{d}{dx}[arcsinu] = \frac{u'}{\sqrt{1-u^2}}$ $\frac{d}{dx}[arccosu] = \frac{-u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}[arctanu] = \frac{u'}{1+u^2}$ $\frac{d}{dx}[arccotu] = \frac{-u'}{1+u^2}$

$\frac{d}{dx}[arcsecu] = \frac{u'}{|u|\sqrt{u^2-1}}$ $\frac{d}{dx}[arccscu] = \frac{-u'}{|u|\sqrt{u^2-1}}$

Proof for $\frac{d}{dx}[arcsinx] =$

3. Differentiate $y = arcsec(2x)$

4. Differentiate $y = 2ln(t^2 + 4) - arctan\frac{t}{2}$

5. Differentiate $y = arcsin(e^{3x})$

6. Find the equation of the tangent line to the graph of the equation $arctan(xy) = arcsin(x + y)$ at (0,0).