

Conic Sections Video Lecture

Sections 10.1-10.4

Course Learning Objectives:

Graph conic sections and solve applications involving them.

Weekly Learning Objectives:

- 1) Know the names of the Conics.**
- 2) Analyze parabolas with vertex at the origin.**
- 3) Analyze ellipses with center at the origin.**
- 4) Analyze ellipses with center at (h,k) .**
- 5) Analyze hyperbolas with center at the origin.**
- 6) Find the asymptotes of a hyperbola.**
- 7) Analyze hyperbolas with center at (h,k) .**
- 8) Solve applied problems involving conic sections.**

Conic Sections

Conic sections are the curves formed by the intersection of a plane with the two nappes of a cone.

We have already studied parabolas from an algebraic point of view. We know that the equation $f(x) = ax^2 + bx + c$ has a set of solutions that lie on a \cup shape. We can determine, depending on the values of a , b , and c , whether the parabola opens up or down, where the vertex is, where the intercepts are, as well as finding additional points on the graph.

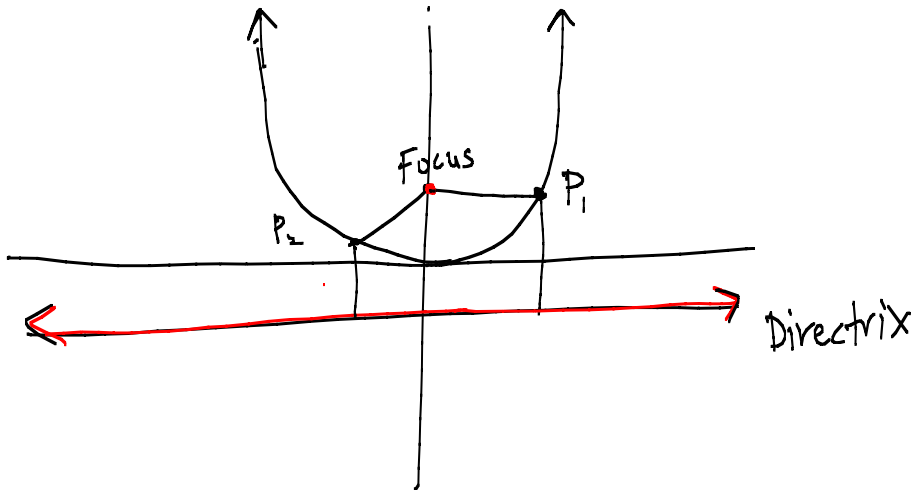
$$a > 0$$

$$a < 0$$

Vertex Formula: $(-\frac{b}{2a}, f(-\frac{b}{2a}))$

We will now define the conic sections from a geometric point of view and show that these definitions are consistent with the algebraic discussion that we have had previously.

A parabola is the set of points equidistant from a given point (focus) and a given line (directrix).

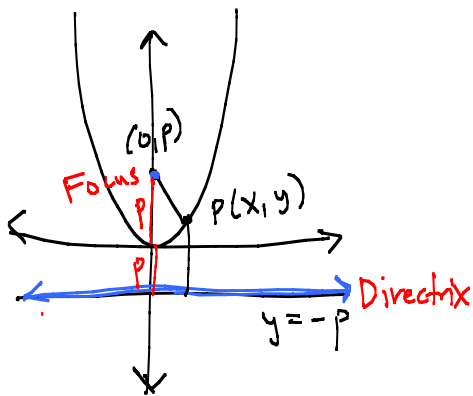


Derivation of the equation of a parabola with a vertical axis.

Start with a parabola centered at the origin. Let $P(x, y)$ be any point on the parabola, then the distance between P and the focus F is:

Distance from P to the directrix is:

By the definition of a parabola, these distances must be equal:



Parabola with vertical axis

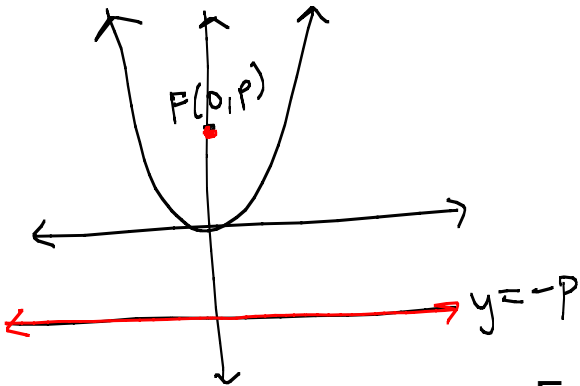
$$x^2 = 4py \text{ or } y = \frac{1}{4p}x^2$$

Vertex $V(0, 0)$

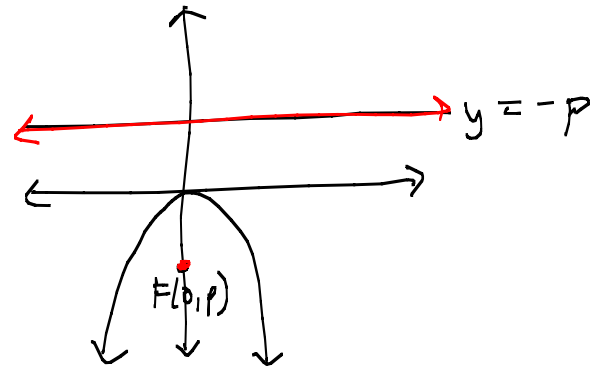
Focus $F(0, p)$

Directrix $y = -p$

If $p > 0$, opens up



If $p < 0$, opens down



Focal Diameter = $4p$

Parabola with horizontal axis

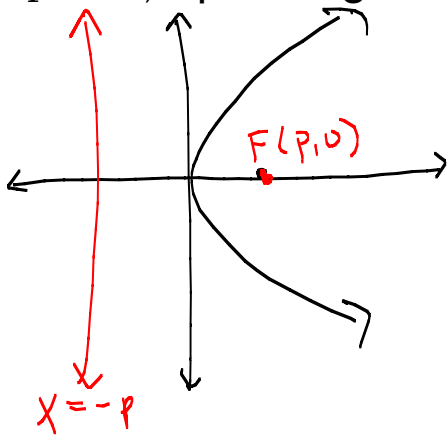
$$y^2 = 4px \text{ or } x = \frac{1}{4p}y^2$$

Vertex $V(0, 0)$

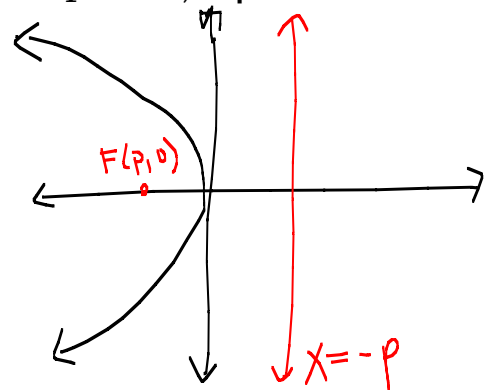
Focus $F(p, 0)$

Directrix $x = -p$

If $p > 0$, opens right



If $p < 0$, opens left



Focal Diameter = $4p$

Application: If you put a light source at the focus, then light rays would reflect off of a parabola. Headlights work this way.

Examples:

1. Find focus, directrix, and focal diameter of the parabola and sketch the graph.

a) $y = -2x^2$

b) $16x - y^2 = 0$

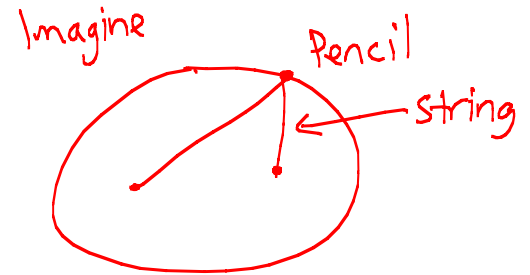
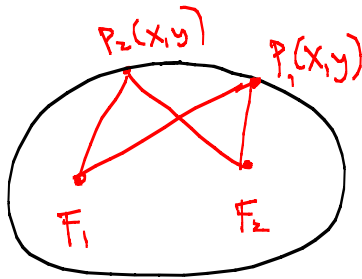
2. Find the equation of a parabola that satisfies the following conditions.

a) vertex at origin, Focus at $(5, 0)$

b) vertex at origin, Equation of Directrix: $y = 6$

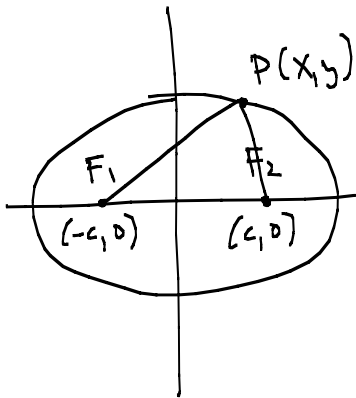
c) Focal diameter 8, vertex at origin, Focus on the negative y -axis.

An ellipse is the set of all points in the plane the sum of whose distances from two fixed points F_1 and F_2 is a constant. These two points are called the foci (plural of focus) of the ellipse.



Derivation of formula of ellipse.

Let the sum of the distances from a point on the ellipse to the foci be $2a$, then:



Ellipse centered at the origin

Equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a > b > 0 \quad (a \text{ is bigger than } b)$$

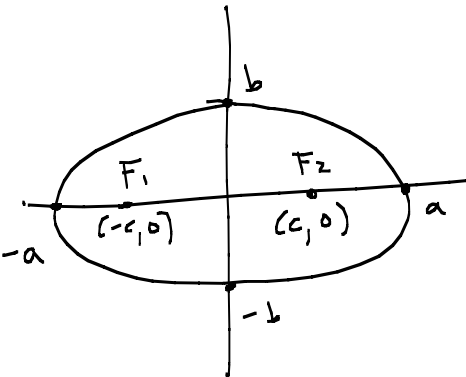
Vertices: $(\pm a, 0)$

Major Axis: Horizontal, length $2a$

Minor Axis: Vertical, length $2b$

Foci: $(\pm c, 0)$ $c^2 = a^2 - b^2$

Graph:



Equation:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$a > b > 0$$

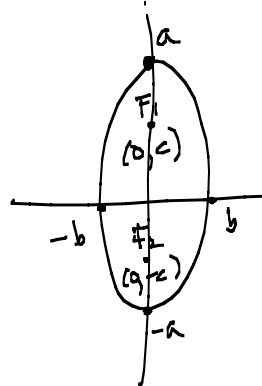
Vertices: $(0, \pm a)$

Major Axis: Vertical, length $2a$

Minor Axis: Horizontal, length $2b$

Foci: $(0, \pm c)$ $c^2 = a^2 - b^2$

Graph:



Examples:

3. Find the vertices, foci, length of major and minor axis, and sketch the graph for each of the following:

a) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

b) $20x^2 + 4y^2 = 5$

c) $\frac{(x+2)^2}{4} + y^2 = 1$

d) $x^2 + 4y^2 + 20x - 40y + 300 = 0$

e) $x^2 + 4y^2 + 20x - 40y + 100 = 0$

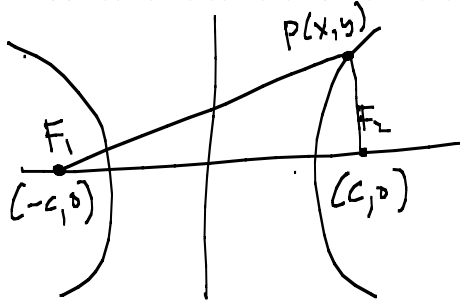
4. Write the equation of an ellipse satisfying the following conditions.

a) Foci at $(\pm 4, 0)$, vertices $(\pm 5, 0)$

b) Foci at $(0, \pm 5)$, length of major axis 12

- c) Endpoints of the minor axis $(0, \pm 3)$, distance between foci is 6.

A hyperbola is the set of all points in the plane, the difference of whose distances from two fixed points F_1 and F_2 is a constant. The two fixed points are called the foci of the hyperbola.



Derivation of formula of hyperbola

Let the Foci be at $(\pm c, 0)$ and let the difference between the distances from the two foci is $\pm 2a$, then:

Hyperbola

Equation:

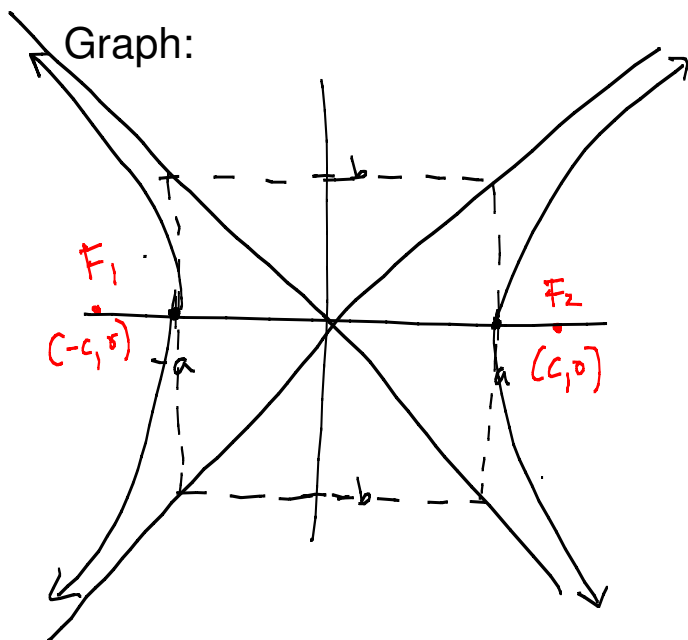
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (x \text{ first})$$

Vertices: $(\pm a, 0)$

Transverse Axis: Horizontal, length $2a$

Asymptotes: $y = \pm \frac{b}{a}x$

Foci: $(\pm c, 0)$ $c^2 = a^2 + b^2$



Equation:

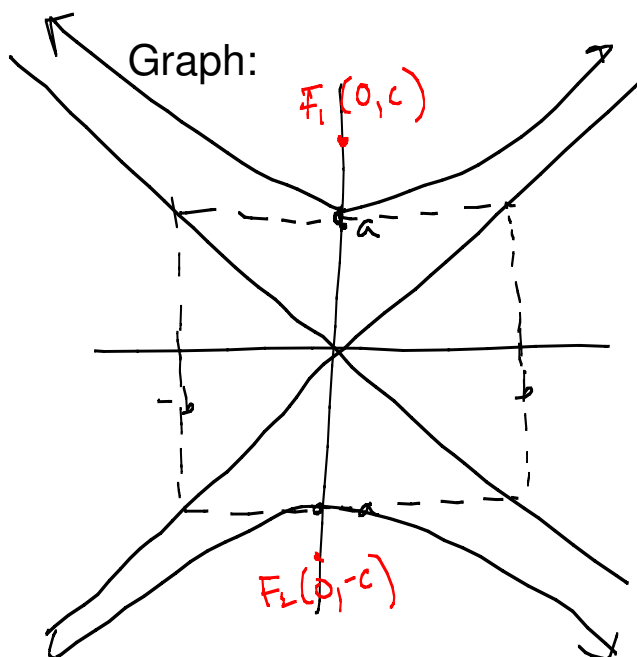
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad (y \text{ first})$$

Vertices: $(0, \pm a)$

Transverse Axis: Vertical, length $2a$

Asymptotes: $y = \pm \frac{a}{b}x$

Foci: $(0, \pm c)$ $c^2 = a^2 + b^2$



To graph:

- 1) Sketch the central box
- 2) Sketch the asymptotes
- 3) Plot vertices
- 4) Sketch 2 branches

Examples:

5. Find the vertices, foci, and asymptotes of the hyperbola and sketch its graph.

a) $\frac{y^2}{9} - \frac{x^2}{16} = 1$

b) $x^2 - 2y^2 = 3$

c) $\frac{(y - 1)^2}{25} - (x + 3)^2 = 1$

d) $x^2 - y^2 = 10x - 10y + 1$

6. Write the equation of a hyperbola satisfying the following conditions.

a) Foci $(0, \pm 10)$, vertices $(0, \pm 8)$

b) Vertices $(0, \pm 6)$, asymptotes $y = \pm \frac{1}{3}x$