

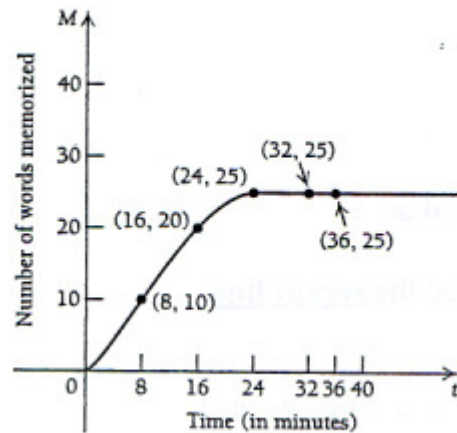
Average Rates of Change

The difference between Average Rate of Change and Instantaneous Rate of Change:

This section focuses on average rates of change only.

Example:

The total number of words $M(t)$ that a person can memorize in time t , in minutes, is shown in the following graph.



Find the average rate of change of M as t changes from:

0 to 8

8 to 16

16 to 24

24 to 32

32 to 36

What happens to the average rate of change after 24 minutes, and why?

Let's formalize this idea of average rate of change:

Given a function $f(x)$ and we want to find the average rate of change of y with respect to x between the points $P(x_1, f(x_1))$ and $Q(x_2, f(x_2))$:

The average rate of change of $f(x)$ between x_1 and x_2 :

The line between $P(x_1, y_1)$ and $Q(x_2, f(x_2))$ is called the **secant line**.

Example: Let $f(x) = x^2 + x$.

Find the average rate of change as x changes for :

- $x = 1$ to $x = 2$
- $x = 1$ to $x = 3$
- $x = 2$ to $x = 3$

Suppose that $f(x) = 2x + 3$. What is the average rate of change over any interval?

Alternative notation for finding the average rate of change:

Instead of using the points x_1 and x_2 (subscripts), consider the function $f(x)$ and the points $P(x, f(x))$ and $Q(x + \Delta x, f(x + \Delta x))$. (Usually Δx is something small.)

The difference quotient is:

Example: Let $f(x) = x^2 + x$. Find the difference quotient when $x = 1$ and $\Delta x = 1$ and also when $x = 1$ and $\Delta x = .1$.

If we need to find a difference quotient for many different points, this would get tedious. Instead, we can find a general difference quotient at any x , simplify it, then plug in the point we are interested in.

Example: Find the simplified difference quotient of the function $f(x) = 5x^3$.

Example: Find the simplified difference quotient of the function $f(x) = \frac{4}{x}$.

Example: It is known that an object in free fall will fall a distance s , in feet, in t seconds, given by $s(t) = 16t^2$.

- a) How far does the object fall in 2 secs?
- b) How far does the object fall in 4 secs?
- c) What is the average rate of change of the distance with respect to time during the interval 2 to 4 secs?

Average Velocity: